A Learning-Based Account of Phonological Tiers

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Morphophonological alternations often involve dependencies between adjacent segments. Despite the apparent distance between relevant segments in the alternations that arise in consonant and vowel harmony, these dependencies can usually be viewed as adjacent on a tier representation. However, the tier needed to render dependencies adjacent varies cross-linguistically, and the abstract nature of tier representations in comparison to flat, string-like representations has led phonologists to seek justification for their use in phonological theory. In this paper, I propose a learning-based account of tier-like representations. I argue that humans show a proclivity for tracking dependencies between adjacent items, and propose a simple learning algorithm that incorporates this proclivity by tracking only adjacent dependencies. The model changes representations in response to being unable to predict the surface form of alternating segments—a decision governed by the Tolerance Principle, which allows for learning despite the sparsity and exceptions inevitable in naturalistic data. Tier-like representations naturally emerge from this learning procedure, and, when trained on small amounts of natural language data, the model achieves high accuracy generalizing to held-out test words, while flexibly handling cross-linguistic complexities like neutral segments and blockers. The model also makes precise predictions about human generalization behavior, and these are consistently borne out in artificial language experiments.

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1 Introduction

Phonological theory, and linguistic theory more broadly, often attempts to interpret apparently long-distance dependencies as being adjacent on some representation. For many morphophonological alternations, dependencies are already adjacent in a straight-forward, flat representation. Such is the case for the English plural morpheme, which alternates between [-z] and [-s], matching the stem-final segment's voicing, and separated by [ə] if the stem-final segment is a sibilant, as in (1).

(1) [dɑg-z]
    [kæt-s]
    [hɔrs-əz]

However, vowel and consonant harmony (Rose and Walker, 2004; van der Hulst, 2016) and dissimilation (Bennett, 2013) often involve dependencies between segments that are arbitrarily distant in a flat representation. For instance, the vowels of Turkish suffixes harmonize with the preceding vowel across intervening consonants. The affix vowels in (2) alternate between back {ɑ, ɯ} and front {e, i} to match the backness of the preceding vowel (examples from Nevins 2010:28; Kabak 2011:3).

(2)	[dɑl-lɑr-ɯn]	[jer-ler-in]	[ip-ler-in]
	'branch'-Pl-Gen	'place'-Pl-Gen	'rope'-Pl-Gen

This harmony process can be viewed as a local spreading process across a phonological *tier* containing only the vowels (Goldsmith, 1976; Clements, 1976, 1980). A similar analysis is possible for other alternations. For instance, Aari exhibits sibilant harmony, as exemplified in (3) (Hayward 1990; McMullin 2016:21), where underlying /s/ (3a) surfaces as [ʃ] when preceded by a [−ant] sibilant at any distance (3b). This can be viewed as a local process on a sibilant tier.

(3)	a. /baʔ-s-e/ → [baʔse]	b. /ʔuʃ-s-it/ → [ʔuʃʃit]
	'he brought'	'I cooked'

/ʒaʔ-s-it/ → [ʒaʔʃit]

'I arrived'

/ʃed-er-s-it/ → [ʃederʃit]

'I was seen'

However, the tier needed to render dependencies local varies extensively cross-linguistically. For instance, to view Finnish's vowel harmony as local requires excluding the vowels {i, e} from the tier, as they neither participate in nor block harmony: In (4) (Ringen and Heinämäki, 1999:305), the essive case vowel alternates between back [ɑ] and front [æ] depending on the final harmonizing vowel of the stem (4a), but passes over both consonants and neutral vowels—like [−back] [i] in (4b).

(4) a. [pøytæ-næ]
'table'-Ess
[poutɑ-nɑ]
'fine weather'-Ess

b. [koti-nɑ]
'home'-Ess

In Latin, default /l/ (5a) dissimilates to [r] when preceded by /l/ across varying distances (5b), but the dissimilation is blocked by an intervening /r/ (5c) or an intervening [−cor] consonant (5d) (examples from Cser 2010, organized following McMullin 2016).

(5) a. *nav-alis*
'naval'

b. *popul-aris*
'popular'
lun-aris
'lunar'

c. *flor-alis*
'floral'

d. *pluvi-alis*
'rainy'
leg-alis
'legal'

Consequently, in addition to [l] and [r], the relevant tier must also contain the [−cor] consonants to block the dissimilation (McMullin, 2016; Burness *et al.*, 2021).

Because of the cross-linguistic variation demonstrated by these examples and others (McMullin and Hansson, 2016; Burness, McMullin, and Chandlee, 2021), together with the abstractness of tier representations in comparison to flat representations, phonologists have sought justification for their use in phonological theory. Hayes and Wilson (2008) proposed an *inductive baseline* argument by demonstrating that their phonotactic learner fails to learn nonlocal generalizations unless it is provided a relevant tier projection *a priori*. Hayes and Wilson viewed these results as learning-based evidence in favor of the use of tiers in phonological theory. Formal-language-theoretic results demonstrate that restricting a learner's hypothesis space to tier-based generalizations allows for proving strong, theoretical learning results (Heinz, Rawal, and Tanner, 2011; McMullin, 2016; Burness and McMullin, 2019, 2021), which provides theoretical support for Hayes and Wilson (2008)'s empirical results. Goldsmith and Riggle (2012) demonstrated that tier representations allow for better compression of linguistic data, and viewed this as statistical justification for the use of tier representations.

In this paper, I seek to approach this important question from the opposite direction, by building a computational model bottom-up (section 2) from an independently-motivated psychological mechanism—namely, results from sequence learning experiments, which forcefully demonstrated that learners more readily track dependencies between adjacent items (Saffran, Aslin, and Newport 1996; Saffran *et al.* 1997; Aslin, Saffran, and Newport 1998) than nonadjacent items (Santelmann and Jusczyk, 1998; Gómez, 2002; Newport and Aslin, 2004; Gómez and Maye, 2005) (see section 2.1). I demonstrate that attention being focused on adjacent dependencies motivates the removal of any segments preventing prediction of the alternating segment's surface form, effectively creating a new representation, which can be interpreted as a tier. This process mirrors the iterative erasure mechanism proposed by Jardine and Heinz (2016) and Burness and McMullin (2019, 2021) as a natural consequence of the formal properties of Tier-Strictly-2-Local languages and functions.

This convergent development from different perspectives supports an iterative change of representation as a possible mechanism involved in phonological learning. Moreover, my model uses the Tolerance Principle (Yang, 2016) to determine when a change of representation is needed, which improves the model's robustness against the exceptions and sparsity of naturalistic data. In section 3, I discuss prior models for learning nonlocal phonological dependencies, and in section 4 compare the proposed model—and prior models—to human behavior on multiple prior artificial language experiments. The proposed model is the only one to match human behavior in all cases. In section 5, I evaluate the model at learning natural language, nonlocal alternations. The model effectively learns Turkish vowel harmony, Finnish vowel harmony, and Latin liquid dissimilation, with substantially greater test accuracy than prior models. These results demonstrate that the model can handle parasitic harmony (Turkish secondary rounding harmony), neutral segments (Finnish vowel harmony), and blocking segments (Latin liquid dissimilation). The model achieves these results on datasets characteristic of the size of children's vocabularies at the time they appear to begin extending harmony to novel words (Altan, 2009). Because of the model's basis in an independently-motivated psychological mechanism, I argue that these results provide a possible learning-based account of tier representations.

2 Model Description

My model, which I call D2L for *Distant To Local*, is based on experimental results in sequence learning, which have demonstrated that learners track adjacent dependencies more readily than nonadjacent dependencies. I review these results in section 2.1.

The input to D2L is a set, V, of (UR, SR) input-output pairs. I follow the commonly-adopted position that learners only construct abstract URs when doing so is necessary to account for surface alternations (Kiparsky, 1968; Peperkamp *et al.*, 2006; Ringe and Eska, 2013; Tesar, 2013; O'Hara, 2017; Richter, 2018, 2021; Belth, 2023, To appear), and I treat segments that alternate on the surface as underlyingly underspecified. This is consistent with Nevins (2010)'s treatment of harmony as an alternating segment searching for a value, and Richter (2018, 2021); Belth (2023, To appear) provide accounts of how surface alternation can be used to construct such URs. Future work will focus on the important problem of jointly learning URs and processes mapping between URs and SRs (Cotterell, Peng, and Eisner 2015; Hua, Jardine, and Dai 2020; Ellis *et al.* 2022; Belth 2023).

D2L attempts to form a generalization to predict the surfacing of the alternating segments, A, in terms of segments adjacent to them. If this fails, D2L deletes the adjacent segments that failed to account for the alternation and tries again. This process iterates until an adequate generalization is discovered or until no more segments can be deleted. In section 2.2, I describe the structure of D2L's generalizations: how they project a tier and how the surface form of alternating segments is predicted from adjacent segments. Then in section 2.3, I describe how D2L automatically constructs such generalizations.

The code is publicly available at https://github.com/cbelth/Learning-Based-Tiers.

2.1 Experimental Studies of Sequence Learning

Studies of statistical sequence learning have studied whether learners can segment a continuous speech stream into discrete units based on statistical information (Saffran, Aslin, and Newport 1996; Saffran *et al.* 1997; Aslin, Saffran, and Newport 1998). These studies hypothesized that infants could track *transitional probabilities*, which capture the strength of adjacent dependencies in sequential data, and use them as a cue to segment speech. The studies found infants as young as 8-months to be sensitive to dependencies between adjacent syllables. The ability to track adjacent dependencies has also been attested between morphemes (Santelmann and Jusczyk, 1998), nonlinguistic tones (Saffran *et al.*, 1999), and visual shapes (Fiser and Aslin, 2002). The diversity of types of elements for which this ability has been observed suggests that the ability to track adjacent dependencies may be

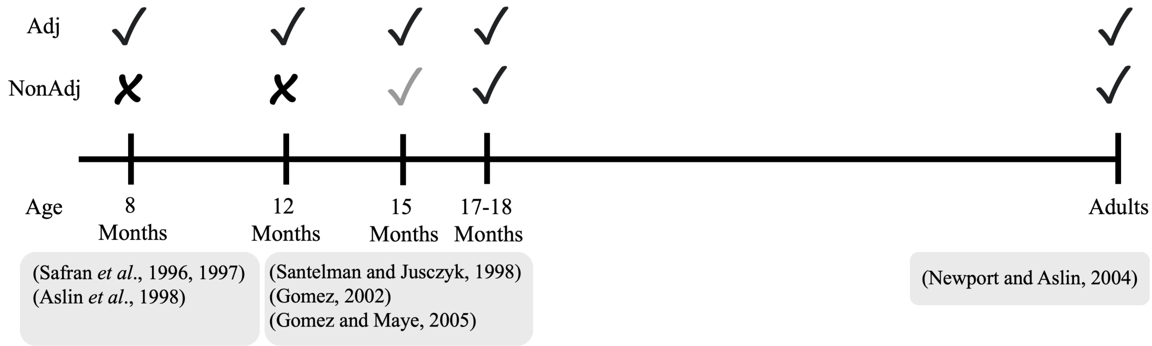

Figure 1: The asymmetrical developmental trajectory of learners' ability to track adjacent and nonadjacent dependencies. Infants as young as 8mo old show evidence of being able to track adjacent dependencies; this ability persists into adulthood. In contrast, learners do not show evidence of tracking nonadjacent dependencies until around 15mo.

neither limited to a particular kind of linguistic structure nor even the domain of language.

Statistical learning studies have also been directed towards nonadjacent dependencies. While the ability to track adjacent dependencies has been widely attested even for infants as young as 8 months old, Santelmann and Jusczyk (1998) found no evidence of learners tracking nonadjacent dependencies even at 15-months-old.

The ability to track nonadjacent dependencies does eventually emerge. Adults show a sensitivity to dependencies between nonadjacent phonological segments (Newport and Aslin, 2004), and 18-month-old children show sensitivity to dependencies between both nonadjacent morphemes (Santelmann and Jusczyk, 1998) and words (Gómez, 2002). Gómez and Maye (2005) attempted to map the developmental trajectory of this ability to track non-adjacent dependencies, and found that it grew gradually with age. At 12 months, children showed no evidence of tracking nonadjacent dependencies, but they began to do so by 15 months, and showed further advancement at 17 months.

Thus, the developmental trajectory of adjacent and nonadjacent dependencies, shown in figure 1, is asymmetrical. Children as young as 8-months-old show evidence of tracking adjacent dependencies, and this ability persists into adulthood, but children only begin to show evidence of tracking nonadjacent dependencies at around 15 months.

Even as sensitivity to nonadjacent dependencies develops, learners still more readily track local dependencies. Gómez (2002) found that 18mo-olds could track nonadjacent dependencies, but that they only did so when adjacent dependencies were unavailable. Gómez and Maye (2005) replicated these results with 17mo-olds and described the situation like this (p. 199): "It is as if learners are attracted by adjacent probabilities long past the point that such structure is useful." Indeed, artificial language experiments have repeatedly demonstrated that learners more easily learn local phonological processes than nonlocal ones (Baer-Henney and van de Vijver, 2012), and, when multiple possible phonological generalizations are consistent with exposure data, learners systematically construct the most local generalization (Finley, 2011; White *et al.*, 2018; McMullin and Hansson, 2019).

The evidence that infants track adjacent dependencies across a range of items not limited to the linguistic domain, that the ability to track nonadjacent dependencies appears to emerge later in development than that for adjacent dependencies, and that learners only show evidence of tracking nonadjacent dependencies as a last resort when adjacent dependencies fail them, strongly supports the conclusion that human's exhibit a proclivity for adjacency. This proclivity constitutes an independent foundation for the proposed model.

2.2 The Structure of Generalizations

D2L constructs a local rule, from which the surface form of alternating segments (the rule's target) can be predicted from left-adjacent or right-adjacent segments after a (possibly empty) set of segments has been deleted. Consequently, I characterize a generalization as the composition of a local rule, r_{adj}, with a tier projection, $\text{proj}(x, T)$, which preserves only the segments in input sequence $x = x_i, \ldots, x_n$ that are contained in the tier $T \subseteq \Sigma$ (6).

(6) $g(x) = r_{\text{adj}} \circ \text{proj}(x, T)$

Clearly, the $\text{proj}(\cdot, T)$ can interchangeably be viewed as creating a new sequence in which only segments in T are preserved, or as creating a new sequence in which all segments not in T are deleted. Consequently, it will sometimes be useful to refer to the complement of T as the **deletion set** $D \triangleq \Sigma \setminus T$. Thus, tier projection is an *erasing function* (Heinz, Rawal, and Tanner, 2011), which creates a new sequence $t_1, \ldots, t_m$, where every segment $x_i \notin T$ is deleted. The tier-segments are annotated with their indices in the original sequence so that the results of the rule application can be written to the correct position in the output sequence. For instance, a [+sib][1] tier projection is shown in (7) for a hypothetical input. I use <·> to denote a tier projection. The superscripts denote the annotation of each sibilant's position in the input sequence.

(7) proj(/ʃoku-s-is/, [+sib]) = <ʃ$^{(1)}$s$^{(5)}$s$^{(7)}$>

Once the tier is projected, the local rule r_{adj} applies over it. This r_{adj} has the structure of an *SPE* rule (Chomsky and Halle, 1968) with either a left (8a) or a right context (8b).

(8) a. $A \rightarrow B$ / C __

b. $A \rightarrow B$ / __ C

In this paper, $A \rightarrow B$ can either be Agree(A, F), which sets A's features $f \in F$ to match those in C, or Disagree(A, F), which sets them to the opposite of C's. This follows Nevins 2010 in drawing analogy to syntactic Agree (Chomsky, 2001a,b). I discuss this point further in the appendix. This brings the rule schemas to those in (9).

(9) a. Agree(A, F) / C __

Agree(A, F) / __ C

b. Disagree(A, F) / C __

Disagree(A, F) / __ C

An example sibilant harmony rule is (10).

(10) $g(x) = \text{Agree}([\text{+sib}], \{\text{ant}\}) / [\text{+sib}]\_\_ \circ \text{proj}(x, [\text{+sib}])$

Because harmony patterns tend to spread (Nevins, 2010; Burness, McMullin, and Chandlee, 2021), the rules apply iteratively.[2] If the rule has a left-context, it applies left-to-right; otherwise it applies right-to-left.

The output sequence is generated by copying every nontier element to the output directly, and copying every tier element to the output position corresponding to its annotated index. Example (11) shows the sibilant rule (10) applying to a hypothetical input—underlines show the rule applications.

(11) $\text{/ʃoku-s-is/} \rightarrow < \text{ʃ}^{(1)}\text{s}^{(5)}\text{s}^{(7)} > \rightarrow < \underline{\text{ʃ}^{(1)}\text{ʃ}^{(5)}}\text{s}^{(7)} > \rightarrow < \text{ʃ}^{(1)}\underline{\text{ʃ}^{(5)}\text{ʃ}^{(7)}} > \rightarrow \text{[ʃokuʃiʃ]}$

The restriction of operations to Agree and Disagree and the context to either a single position to the left or right is to keep the model and its discussion succinct. To handle epenthesis, deletion, and contexts larger than a single segment (e.g., intervocalic voicing), D2L could be placed in a broader phonological learning framework like Belth (To appear)'s.

Since the rules are applied iteratively and invoke a tier-local left or right context, they relate to Output Tier-based Strictly 2-Local functions (Burness and McMullin, 2019; Burness, McMullin, and Chandlee, 2021). Thus, the algorithm I describe in section 2.3, which is grounded in a sensitivity to adjacent dependencies (section 2.1), attempts to provide bottom-up support for that class of functions. The appendix includes discussions of connections with search-and-copy accounts of vowel harmony, and possible connections to syntactic Agree.

2.2.1 Examples and Expressivity

These tier-based generalizations can express a wide range of behaviors, including *transparency, parasitic harmony*, and *blocking*.

Transparent segments do not participate in an alternation. For example, as described in section 1, the Finnish vowels {i, e} are transparent in vowel harmony (4). This can be expressed by excluding the neutral vowels from the tier T = [−cons] \ {i, e}. Since all tier vowels harmonize, and r_{adj} applies over the tier projection, the context can be C = [−cons]. This is stated in (12), where A = [?back] targets vowels unspecified for [back], causing them to harmonize with a vowel to the left on the tier, which includes all vowels except {i, e}.

(12) Agree([?back], {back}) / [−cons] __ ∘ proj(·, [−cons] \ {i, e})

For example, if the Ess suffix vowel in [ko̲tinɑ̲] 'home'-Ess from (4b) is underlyingly underspecified for [back], rule (12) would derive the surface form as in (13). Here /A/ is the /−round,+low,?back/ vowel, which alternates between [+back] [ɑ] and [−back] [æ].

(13) /koti-nA/ $\rightarrow$ $<\text{o}^{(2)}\text{A}^{(6)}>$ $\rightarrow$ $<\underline{\text{o}^{(2)}\text{ɑ}^{(6)}}>$ $\rightarrow$ [ko̲tinɑ̲]

In *parasitic harmony*, segments only harmonize with respect to some feature when they agree in another feature. In Kachin Khakass, only [+high] {i, ɯ, y, u} suffixal vowels undergo rounding harmony, and only with [+high] stem vowels (14a). They fail to harmonize with [−high] {e, ø, a, o} stem vowels (14b) and [−high] suffixal vowels never harmonize (14c). Data from Korn 1969; Burness, McMullin, and Chandlee 2021:18.

(14)	a.	b.	c.
	[ky̲n-ny̲]	[ok-tɯn]	[kyn-ge]
	'day'-Acc	'of the arrow'	'to the day'
	[ku̲ʃ-tu̲n]		[pol-za]
	'of the bird'		'if he is'

This can be captured by including only [+high] vowels on the tier: T = [+high, −cons] as in (15). The context can again be [−cons] since the tier already excludes [−high] vowels.

(15) Agree([?round], {round}) / [−cons] __ ∘ proj(·, [+high, −cons])

In some cases, such as Turkish secondary rounding harmony (see section 5.1), [+high] vowels undergo rounding harmony with vowels of any height. This can be expressed by including all vowels on the tier, while excluding [−high] vowels from the target (16).[3]

(16) $\text{Agree}([-\text{high}, ?\text{round}], \{\text{round}\}) / [-\text{cons}] \_\_ \circ \text{proj}(\cdot, [-\text{cons}])$

In some cases, harmony or dissimilation is *blocked* by some segments. For example, in Khalkha Mongolian (Nevins, 2010:137) (17) the rounding harmony in (17a) is blocked by the [+round] vowels {u, ʊ} (17b).

(17) a. [to̲r-o̲ːd]
'be.born'-PERF
[ɔ̲r-ɔ̲ːd]
'enter'-PERF

b. [tor-uːl-eːd]
'be.born'-CAUS-PERF
[ɔr-ʊːl-aːd]
'enter'-CAUS-PERF

Critically, the PERF affix vowels {e, a} in (17b) are [−round], implying that they do not harmonize opaquely with the [+round] {u, ʊ} blockers. This blocking can be expressed by including [+round] on the tier, but excluding [+round] from the context (18).

(18) $\text{Agree}([?\text{round}], \{\text{round}\}) / [-\text{round}] \_\_ \circ \text{proj}(\cdot, [-\text{cons}])$

This must be combined with a default [−round] value to account for [−round] vowels surfacing in (17b) when harmony is blocked. I discuss discovering defaults in section 2.3.3.

2.3 Learning

D2L follows the steps in (19). The cases of assimilation and dissimilation are symmetrical. For clarity, I describe D2L in the context of assimilation, and discuss in section 2.3.4 how D2L automatically infers whether the alternation is assimilatory or dissimilatory.

(19) **Input:** (UR, SR) pairs V and a set of segments A that alternate on features F

1. Initialize tier $T = \Sigma$ (equivalently deletion set $D = \emptyset$)

2. **While** $T \neq \emptyset$ **do**

3. – $g_l = \text{Agree}(A, F)/C_l\_\_ \circ \text{proj}(\cdot, T)$

4. – $g_r = \text{Agree}(A, F)/\_\_C_r \circ \text{proj}(\cdot, T)$

5. – $g = \arg\max_{g \in \{g_l, g_r\}} \text{acc}(g, V)$ (§ 2.3.1)

6. – **If** $\text{sat}(g, V)$ **then** ▹ Checks if rule is sufficiently accurate (§ 2.3.1)

7. — **Return** g

8. – Remove from T segments adj. to A on T that cannot account for alternation (§ 2.3.2)

The input to D2L is a set of input-output (UR, SR) pairs, such as (20a), where I use /S/ for alternating sibilants.[4] The alternating segments, A, and what features they alternate on, F, are directly computed from discrepancies between these inputs and outputs (20b). In (20a), since sometimes /S/ → [ʃ] and sometimes /S/ → [s], S $\in A$ and [ant] $\in F$. In my presentation of D2L, I treat alternating segments as underlyingly underspecified (e.g. /S/). I will use (20) as a toy example throughout my presentation of D2L. The (UR, SR) pairs are (20a) and the alternating segments and features are (20b).

(20) a. /ʃoku-S-iS/ → [ʃokuʃiʃ]
/apʃa-S/ → [apʃaʃ]
/ʃun-iS/ → [ʃuniʃ]
/soki-S/ → [sokis]
/sigo-S-iS/ → [sigosis]
/ut-S/ → [uts]

b. $A = \{\text{S}\}$, $F = \{\text{ant}\}$

Initially no segments are deleted, so $T = \Sigma$ (19; step 1). After initialization, D2L enters the while-loop (19; step 2) and constructs left and right rules (19; step 3)-(19; step 4). To do so, the set F is constructed to contain the features that differ across A's surface realizations, and the sets C_l/C_r are constructed to contain every segment tier-adjacent (on the left for g_l and the right for g_r) to an alternating segment. For the words in (20), the first left and right rules are those in (21) because the segments to the left of /S/ are $\{\mathrm{u},\ \mathrm{i},\ \mathrm{a},\ \mathrm{o},\ \mathrm{t}\}$ and to the right of /S/ are $\{\mathrm{i},\ \ltimes\}$, where "$\ltimes$" denotes a right word boundary.

(21) Tier, deletion set, and rules at the first iteration

$T = \Sigma$, $D = \emptyset$

$g_l = \text{Agree}(\{\text{S}\}, \{\text{ant}\})/\{\mathrm{u},\ \mathrm{i},\ \mathrm{a},\ \mathrm{o},\ \mathrm{t}\}\_\_ \circ \text{proj}(\cdot, \Sigma)$

$g_r = \text{Agree}(\{\text{S}\}, \{\text{ant}\})/\_\_\{\mathrm{i}, \ltimes\} \circ \text{proj}(\cdot, \Sigma)$

The accuracy of these rules is then computed (19; step 5) and the more accurate rule is checked to see if it is a sufficiently good generalization (19; step 6)

2.3.1 Computing Accuracy and Rule Quality

The accuracy of a rule g is straight-forwardly defined (22c) as the number of correct predictions made by g over the training instances V (22b), divided by its total number of predictions over the training instances (22a).

(22) a. $n(g, V) \triangleq$ number of g's predictions over V

b. $c(g, V) \triangleq$ number of g's correct predictions over V

c. $\text{acc}(g, V) \triangleq \frac{c(g,V)}{n(g,V)}$

Since rules are applied iteratively, the number of applications and correct applications are computed iteratively as well. Thus, the sibilant harmony rule (23a), which states that underlying /S/ should agree in anteriority with the preceding sibilant (after projecting a [+sib] tier), has two applications (both correct) over the input (23b). I denote rule applications with underlines.

(23) a. $\text{Agree}(\{\text{S}\}, \{\text{ant}\})/[+\text{sib}]\_\_ \circ \text{proj}(x, [+\text{sib}])$

b. /ʃoku-S-iS/ $\rightarrow <ʃ^{(1)}S^{(5)}S^{(7)}> \rightarrow <\underset{\textbf{(+1)}}{\underline{ʃ^{(1)}ʃ^{(5)}}}S^{(7)}> \rightarrow <ʃ^{(1)}\underset{\textbf{(+1)}}{\underline{ʃ^{(5)}ʃ^{(7)}}}> \rightarrow$ [ʃokuʃiʃ]

An application is considered incorrect if either (a) it predicts the incorrect surface form, or (b) the target cannot Agree/Disagree with the contextual segment due to the relevant feature being unspecified. For example, consider rule (24a), which states that underlying /S/ should agree in anteriority with the preceding consonant. This rule will make a correct prediction for /apʃa-S/ → [apʃaʃ] in (24b), since the underlying /S/ taking its anteriority from /ʃ/ indeed leads to it correctly surfacing as [ʃ]. However, for /ʃun-iS/ → [ʃuniʃ], /S/ harmonizing with [+ant] [n] will lead to incorrect surface form [+ant] [s] (24c).

(24) a. $\text{Agree}(\{S\}, \{\text{ant}\}) / [+\text{cons}]\_\_ \circ \text{proj}(\cdot, [+\text{cons}])$

b. /apʃa-S/ $\rightarrow <p^{(2)}ʃ^{(3)}S^{(5)}> \rightarrow <p^{(2)}\underline{ʃ^{(3)}ʃ^{(5)}}> \rightarrow$ [apʃaʃ] ✓

c. /ʃun-iS/ $\rightarrow <ʃ^{(1)}n^{(3)}S^{(5)}> \rightarrow <ʃ^{(1)}\underline{n^{(3)}s^{(5)}}> \rightarrow$ [ʃunis] ✗

A rule like (25a), which states that /S/ should agree in anteriority with the preceding vowel (all segments Σ projected), will produce an error on /apʃa-S/ → *[apʃaS] for the latter reason: /S/ cannot take the feature value for [ant] from /a/, assuming vowels are not specified for consonantal features.

(25) a. $\text{Agree}(\{S\}, \{\text{ant}\}) / [-\text{cons}]\_\_ \circ \text{proj}(\cdot, \Sigma)$

b. /apʃa-S/ $\rightarrow <a^{(1)}p^{(2)}ʃ^{(3)}a^{(4)}S^{(5)}> \rightarrow <a^{(1)}p^{(2)}ʃ^{(3)}\underline{a^{(4)}S^{(5)}}> \rightarrow$ [apʃaS] ✗

Thus, for vocabulary (20), rule (24a) makes a correct prediction for only the sibilants preceded by a consonant that correctly predicts the surface form for /S/. This is shown in (26), where [+ant] = {s, n, p, t} and [−ant] = {ʃ, k, g} ("*" marks errors).

(26) /ʃoku-S-iS/ $\rightarrow <ʃ^{(1)}k^{(3)}S^{(5)}S^{(7)}> \rightarrow <ʃ^{(1)}\underline{k^{(3)}ʃ^{(5)}}S^{(7)}> \rightarrow <ʃ^{(1)}k^{(3)}\underline{ʃ^{(5)}ʃ^{(7)}}>$

→ [ʃokuʃiʃ]

$/\text{apʃa-S}/ \rightarrow \langle \text{p}^{(2)}\text{ʃ}^{(3)}\text{S}^{(5)} \rangle \rightarrow \langle \text{p}^{(2)}\underline{\text{ʃ}^{(3)}\text{ʃ}^{(5)}} \rangle \rightarrow [\text{apʃaʃ}]$

$/\text{ʃun-iS}/ \rightarrow \langle \text{ʃ}^{(1)}\text{n}^{(3)}\text{S}^{(5)} \rangle \rightarrow \langle \text{ʃ}^{(1)}\underline{\text{n}^{(3)}\text{s}^{(5)}} \rangle \rightarrow [\text{ʃuni*s}]$

$/\text{soki-S}/ \rightarrow \langle \text{s}^{(1)}\text{k}^{(3)}\text{S}^{(5)} \rangle \rightarrow \langle \text{s}^{(1)}\underline{\text{k}^{(3)}\text{ʃ}^{(5)}} \rangle \rightarrow [\text{soki*ʃ}]$

$/\text{sigo-S-iS}/ \rightarrow \langle \text{s}^{(1)}\text{g}^{(3)}\text{S}^{(5)}\text{S}^{(7)} \rangle \rightarrow \langle \text{s}^{(1)}\underline{\text{g}^{(3)}\text{ʃ}^{(5)}}\text{S}^{(7)} \rangle \rightarrow \langle \text{s}^{(1)}\text{g}^{(3)}\underline{\text{ʃ}^{(5)}\text{ʃ}^{(7)}} \rangle$

$\rightarrow [\text{sigo*ʃi*ʃ}]$

$/\text{ut-S}/ \rightarrow \langle \text{t}^{(2)}\text{S}^{(3)} \rangle \rightarrow \langle \underline{\text{t}^{(2)}\text{s}^{(3)}} \rangle \rightarrow [\text{uts}]$

There are 8 instances of /S/, and (24a) predicted the correct surface form for 4 of these. Thus, $n(g, V) = 8$, $c(g, V) = 4$, and $\text{acc}(g, V) = 4/8 = 1/2$.

The function $\text{sat}(g, V)$ is a boolean function that returns "True" iff g is satisfactorily accurate over the training data V. In this work, I use the Tolerance Principle (Yang, 2016) as the criterion due to its psychological basis and prior success in computational modeling, lexical, and experimental studies (Schuler, Yang, and Newport 2016; Yang 2016; Richter 2018; Koulaguina and Shi 2019; Emond and Shi 2021; Richter 2021; Belth *et al.* 2021). The Tolerance Principle hypothesizes that learners accept a linguistic generalization when it is computationally more efficient to do so than to reject the generalization. It provides a quantitative, categorical threshold for this tipping point in terms of the generalization's scope and how many exceptions it has (see Yang 2016:ch. 3 for the threshold's derivation). When the threshold is met and a generalization accepted, the exceptions to the generalization can be lexicalized. In the current work, using the Tolerance Principle for evaluating generalizations is achieved—in terms of (22)—via (27).

(27)

$$\text{sat}(g, V) \triangleq n(g, V) - c(g, V) \leq \frac{n(g, V)}{\ln n(g, V)}$$

For the initial rules (21), both g_l and g_r make $n(g_l, V) = n(g_r, V) = 8$ predictions over the vocabulary (20), because there are 8 underlying /S/'s. The left rule in (21) makes only 1 correct prediction: /unt-S/ → [unts]; the remaining 7 predictions err because the underlying /S/'s cannot harmonize with adjacent vowels. The right rule in (21) makes no correct predictions. Thus, $\text{acc}(g_l, V) = 1/8$ and $\text{acc}(g_r, V) = 0/8$. Since the former is more accurate, it is chosen (19; step 5). However, since $8 - 1 > 8/\ln 8$ (i.e., $7 > 3.85$), $\text{sat}(g, V) =$ "False" at (19; step 6), and the tier must be updated.

2.3.2 Updating the Tier

In order for the rule to apply to them, the alternating segments A must always be preserved on the tier. Moreover, any segment currently tier-adjacent to an alternating segment from which the correct surface form cannot be computed cannot be on the tier if the alternation is to be predictable from adjacent dependencies. Consequently, these unuseful segments, which are present in the context sets C_l and C_r, are added to the deletion set D (28).

(28) $D \cup \{s \in C_l \cup C_r :$ agreeing with s is not possible or yields the wrong surface form$\}$

In our example, {u, i, a, o} ∪ {i, ⋉} are added to D. The segment {t} is not added because agreeing /S/ to /t/ correctly yields [s]. For segments that do not occur tier-adjacent to an alternating segment (e.g., /k/) or yield the correct surface form (e.g., [+ant] /t/ adjacent to an /S/ that surfaces as [+ant] [s]), no conclusion is drawn about whether to keep them on the tier or remove them. Thus, D2L takes the smallest natural class that contains all of D but none of A, and removes this from the tier (29) at (19; step 8).

(29) $T \setminus \arg\min_{\{\text{nat class } N : D \subseteq N \wedge A \cap N = \emptyset\}} |N|$

The arg min ranges over natural classes,[5] each of which I refer to with N. The condition $D \subseteq N$ requires that all segments to be deleted (D) are included in the natural class (N), so that they are excluded from the tier. The condition $A \cap N = \emptyset$ requires that no alternating segments (A) are included in N, so that they are preserved on the tier. The arg min returns the smallest natural class satisfying these conditions ($|N|$ is the size of N).[6]

If no such natural class exists, it removes D verbatim, allowing for idiosyncratic tiers that do not fit neatly into a natural class. In our example, both [+cons] and [+sib] include $A = \{\mathrm{S}\}$ and exclude $D = \{\mathrm{u, i, a, o}, \ltimes\}$, so N in (29) ranges over their complements [−cons] and [−sib]. The class [−cons] is smaller (only the vowels) than [−sib] (both vowels and nonsibilant consonants), so [−cons] is deleted; equivalently T = [+cons]. This yields the tier projections shown in (30c), from which the rules (30d) are derived in the second iteration of the while loop.

(30) Deletion set, tier, tier projections, and rules at the second iteration

a. $D = \{\mathrm{u, i, a, o}, \ltimes\}$

b. T = [+cons]

c. /ʃoku-S-iS/ → <ʃ$^{(1)}$k$^{(3)}$S$^{(5)}$S$^{(7)}$>

/apʃa-S/ → <p$^{(2)}$ʃ$^{(3)}$S$^{(5)}$>

/ʃun-iS/ → <ʃ$^{(1)}$n$^{(3)}$S$^{(5)}$>

/soki-S/ → <s$^{(1)}$k$^{(3)}$S$^{(5)}$>

/sigo-S-iS/ → <s$^{(1)}$g$^{(3)}$S$^{(5)}$S$^{(7)}$>

/ut-S/ → <t$^{(2)}$S$^{(3)}$>

d. $g_l = \mathrm{Agree}(\{\mathrm{S}\}, \{\mathrm{ant}\})/\{\mathrm{k, S}, ʃ, \mathrm{n, g, t}\}\_\_ \circ \mathrm{proj}(\cdot, [\mathrm{+cons}])$

$g_r = \mathrm{Agree}(\{\mathrm{S}\}, \{\mathrm{ant}\})/\_\_\{\mathrm{S}, \ltimes\} \circ \mathrm{proj}(\cdot, [\mathrm{+cons}])$

The sets $C_l = \{\mathrm{k, S}, ʃ, \mathrm{n, g, t}\}$ and $C_r = \{\mathrm{S}, \ltimes\}$ are computed from the segments to the left and right of the alternating /S/ on the [+cons] tier. The new rules g_l and g_r again apply to all 8 /S/ segments, so $n(g_l, V) = n(g_r, V) = 8$. The right rule g_r makes 0 correct predictions because /S/ cannot harmonize with "⋉" or a right-adjacent /S/ that also is not specified for anteriority. The left rule makes 4 correct predictions, as shown in (31): 2 on the first word, 1 on the second, and 1 on the last word ("*" marks errors).

(31) /ʃoku-S-iS/ → <ʃ$^{(1)}$k$^{(3)}$S$^{(5)}$S$^{(7)}$> → <ʃ$^{(1)}$$\underline{\text{k}^{(3)}\text{ʃ}^{(5)}}S^{(7)}$> → <ʃ$^{(1)}k^{(3)}$$\underline{\text{ʃ}^{(5)}\text{ʃ}^{(7)}}$>
→ [ʃokuʃiʃ]
/apʃa-S/ → <p$^{(2)}$ʃ$^{(3)}$S$^{(5)}$> → <p$^{(2)}$$\underline{\text{ʃ}^{(3)}\text{ʃ}^{(5)}}$> → [apʃaʃ]
/ʃun-iS/ → <ʃ$^{(1)}$n$^{(3)}$S$^{(5)}$> → <ʃ$^{(1)}$$\underline{\text{n}^{(3)}\text{s}^{(5)}}$> → [ʃuni*s]
/soki-S/ → <s$^{(1)}$k$^{(3)}$S$^{(5)}$> → <s$^{(1)}$$\underline{\text{k}^{(3)}\text{ʃ}^{(5)}}$> → [soki*ʃ]
/sigo-S-iS/ → <s$^{(1)}$g$^{(3)}$S$^{(5)}$S$^{(7)}$> → <s$^{(1)}$$\underline{\text{g}^{(3)}\text{ʃ}^{(5)}}S^{(7)}$> → <s$^{(1)}g^{(3)}$$\underline{\text{ʃ}^{(5)}\text{ʃ}^{(7)}}$>
→ [sigo*ʃi*ʃ]
/ut-S/ → <t$^{(2)}$S$^{(3)}$> → <$\underline{\text{t}^{(2)}\text{s}^{(3)}}$> → [uts]

The successes are when the tier-adjacent consonants {k, ʃ, t} match the surface anteriority of /S/, and the errors are when {n, k, g} do not. Since $4 > 8/\ln(8)$, the rule is still not sufficiently accurate, according to the Tolerance Principle threshold (27). Since the segments {n, k, g} led to incorrect predictions, they are added to *D*, yielding *D* = {u, i, a, o, n, k, g, ⋉}. The natural class [+cons] no longer separates *A* = {S} from *D* = {u, i, a, o, n, k, g, ⋉}, because *D* now contains vowels and consonants. However, [+sib] does separate *A* and *D*, so *T* = [+sib] is set at (19; step 8), yielding (32).

(32) Deletion set and tier at the third iteration
D = {u, i, a, o, n, k, g, ⋉}
T = [+sib]

At the third iteration, the new left rule (33a), which projects a [+sib] tier, correctly predicts all the surface forms (33b) except for /ut-S/, where it fails to apply because there is no stem sibilant (i.e., the rule underextends). When this happens, D2L attempts to infer a default form for the alternating segment, as discussed next in section 2.3.3. The right rule g_r will have zero accuracy for the same reason as the prior iteration, so the left rule is evaluated under the Tolerance Principle at (19; step 6). Since g_l applies to only the first 7 instances of /S/ (the 8th being handled by the default case, as discussed next), $n(g_l, V) = 7$.

As shown in (33b), the rule predicts the correct surface form in all 7 cases, so $c(g_l, V) = 7$. Since $0 \leq 7/\ln 7$, this rule is accepted and returned[7] (19; step 7).

(33) Successful rule and its predictions at the third iteration

a. $g_l = \text{Agree}(\{\text{S}\}, \{\text{ant}\})/\{\text{s}, \text{ʃ}\}\_\_ \circ \text{proj}(\cdot, [\text{+sib}])$

b. $\text{/ʃoku-S-iS/} \rightarrow <\text{ʃ}^{(1)}\text{S}^{(5)}\text{S}^{(7)}> \rightarrow <\underline{\text{ʃ}^{(1)}\text{ʃ}^{(5)}}\text{S}^{(7)}> \rightarrow <\text{ʃ}^{(1)}\underline{\text{ʃ}^{(5)}\text{ʃ}^{(7)}}> \rightarrow [\text{ʃokuʃiʃ}]$

$\text{/apʃa-S/} \rightarrow <\text{ʃ}^{(3)}\text{S}^{(5)}> \rightarrow <\underline{\text{ʃ}^{(3)}\text{ʃ}^{(5)}}> \rightarrow [\text{apʃaʃ}]$

$\text{/ʃun-iS/} \rightarrow <\text{ʃ}^{(1)}\text{S}^{(5)}> \rightarrow <\underline{\text{ʃ}^{(1)}\text{ʃ}^{(5)}}> \rightarrow [\text{ʃuniʃ}]$

$\text{/soki-S/} \rightarrow <\text{s}^{(1)}\text{S}^{(5)}> \rightarrow <\underline{\text{s}^{(1)}\text{s}^{(5)}}> \rightarrow [\text{sokis}]$

$\text{/sigo-S-iS/} \rightarrow <\text{s}^{(1)}\text{S}^{(5)}\text{S}^{(7)}> \rightarrow <\underline{\text{s}^{(1)}\text{s}^{(5)}}\text{S}^{(7)}> \rightarrow <\text{s}^{(1)}\underline{\text{s}^{(5)}\text{s}^{(7)}}> \rightarrow [\text{sigosis}]$

$\text{/ut-S/} \rightarrow <\text{S}^{(3)}> \rightarrow [\text{ut*S}]$

2.3.3 Default Values

When a candidate rule underextends, D2L takes the set of alternating segments that the rule does not account for and computes the set of surface realizations of those underextensions. If they do not alternate, the surface form is taken as the default. For instance, the rule (33a) underextends for the affix sibilant in /ut-S/, which surfaces as [s] (33b). Taking [s] as the default form for /S/ works, since there are no underextensions of (33a) where /S/ surfaces as anything else. If there were such underextensions, D2L would reject the candidate rule and continue the while-loop.

As another example, when Finnish stems contain only neutral vowels, alternating affix vowels are usually [−back] by default (Ringen and Heinämäki, 1999). For example, the essive affixal vowel, which alternated between [+back] [ɑ] and [−back] [æ] in (4) surfaces as [−back] [æ] when the stem contains only the neutral vowel [e] (34) (Nevins, 2010:76).

(34) [velje-næ]

'road'-Ess

A [back] harmony rule like (12), which excludes neutral vowels {i, e} from the tier, will underextend to words like (34) with only neutral vowels. However, because these underextensions consistently surface as [−back] vowels, D2L infers this as the default.

2.3.4 Assimilation vs. Dissimilation

From observing a segment that alternates, a learner will not immediately know whether the alternation is due to assimilation or dissimilation. However, attempting to account for an assimilatory alternation by dissimilating from something in the phonological environment (or vice versa) is highly unlikely to yield a productive generalization. Thus, figuring out whether an alternation is assimilatory or dissimilatory should not present a serious challenge to learning. Consequently, the D2L algorithm (19) runs twice in parallel—one searching for an assimilatory rule and one searching for a dissimilatory rule. When searching for an assimilatory rule, g_l and g_r are constructed with Agree, and when searching for dissimilatory rule, they are constructed with Disagree. All other aspects are identical. In most conceivable cases, only one search will yield a productive generalization, in which case the generalization from the successful search is chosen. In the unlikely case that both searches yield a generalization, the more accurate one is chosen. This scenario never occurs in any experiments.

The reason it is necessary to take this approach, instead of expanding (19; step 3)-(19; step 4) to include two more rules (i.e., left and right dissimilatory rules), is that it is not possible to maintain a single deletion set for both assimilation and dissimilation. In assimilation, the segments to delete are those that assimilating with yields the wrong surface form; in dissimilation they are those that dissimilating from yields the wrong surface form. Running D2L twice in parallel allows for maintaining these two, distinct deletion sets.

2.4 Strict Locality as a Special Case

As is well-recognized in the literature, D2L's use of tiers unifies learning both local and nonlocal alternations (e.g., Heinz, Rawal, and Tanner 2011; Jardine and Heinz 2016; McMullin 2016). Because it starts with an empty deletion set, a strictly-local alternation—one determined by string-adjacency—will be discovered on the first iteration of the algorithm.

3 Prior Models

Most prior models for learning nonlocal phonological generalizations, with the exception of Burness and McMullin (2019, 2021)'s models, have focused on phonotactics, whereas D2L is focused on alternations. I group prior models into statistical (section 3.1), formal-language-theoretic (section 3.2), and neural network (section 3.3) models.

3.1 Statistical Models

Hayes and Wilson (2008) demonstrated that a phonotactic learner sensitive to only fixed-length sequences failed to learn constraints for long-distance patterns. But, if provided a projection of the data onto a relevant tier, the model was able to learn relevant phonotactic constraints on that tier. However, the model did not learn what tier to project. Gouskova and Gallagher (2020) extended the Hayes and Wilson 2008 model to automatically learn projections. The authors observed that many nonlocal dependencies, despite being arbitrarily far away in principle, often occur within a window of three segments (i.e., a trigram). Their model uses Hayes and Wilson 2008 to extract baseline phonotactic constraints. Some of these are trigram constraints of the form *X[]Y, where X and Y are sets of segments, and [] allows any segment to intervene. Gouskova and Gallagher (2020)'s model then uses these trigram constraints to project a tier, over which additional constraints are learned. In this respect, GG is like D2L, which also first tracks string-local dependencies. But GG tracks nonadjacent X's and Y's and learns phonotactics, not alternations. GG preserves both X and Y on the tier, by setting it to the smallest natural class that contains the segments from both X and Y. For example for the data in (20a)—surface forms reproduced in (35)—if the absence of nonharmonizing sibilant trigrams is sufficiently statistically conspicuous, then the Hayes and Wilson 2008 model may learn the constraints *[s][][ʃ] and *[ʃ][][s].

(35) [ʃokuʃiʃ]
[apʃaʃ]
[ʃuniʃ]
[sokis]
[sigosis]
[uts]

The smallest natural class containing both [s] and [ʃ] is [+stri], so Gouskova and Gallagher (2020)'s model would then project the [+stri] tier and reapply Hayes and Wilson 2008 over that projection. The success of this model depends upon the trigram restriction being clear enough in the data to discover the relevant projection, and upon the effectiveness of Hayes and Wilson 2008 at discovering constraints over the projection. Moreover, the authors recognized a limitation: the model's inability to capture more complex phenomena like opaque or blocking segments, which must be included on the tier but do not participate in the restriction. This is because the model constructs the tier based on the sets X and Y, which do not contain information about blocking segments.

Goldsmith and Riggle (2012) proposed an information theoretic model for justifying a tier-based descriptive account of Finnish vowel harmony. They used Goldsmith and Xanthos (2009)'s Hidden Markov Model (HMM) approach to extract the two classes of segments that maximize the probability of the data. Because the segments of a word tend to alternate between consonants and vowels (e.g. CVCV is much more frequent than CCVV), this HMM approach is best-suited for extracting the two categories *consonant* and *vowel*.

Goldsmith and Riggle (2012) used a Boltzmann model to score phonological illformedness in terms of unigram probabilities and bigram mutual information over the surface string and the vowel tier. The intuition for the model is that it combines the frequency of segments (unigrams) with the frequency of bigrams over the words and vowel-tier projections to score phonological illformedness. This model is largely limited to interactions between all vowels or all consonants, because the HMM usually constructs the consonant and vowel categories. Thus, on (35), the model could find dependencies between sibilants on the consonant tier, but nonsibilant consonants would prevent some sibilants from occurring within the purview of the bigram mutual-information computation.

3.2 Formal-Language-Theoretic Approaches

Heinz (2010) proposed a model for learning long-distance phonotactics based on the *precedence* relation. However some long-distance patterns cannot be accounted for in terms of the precedence relation (Heinz, 2010; Jardine and Heinz, 2016). Consequently, later formal-language-theoretic approaches have instead targeted the class of Tier Strictly-Local (TSL) constraints (Heinz, Rawal, and Tanner, 2011), which have been argued to subsume most or all nonlocal consonant interactions (McMullin, 2016). Their functional analogue, the TSL functions, are argued to cover a broad range of nonlocal processes (Burness, McMullin, and Chandlee, 2021).

In particular, Jardine and Heinz (2016) proposed a model for learning Tier-based Strictly 2-Local (TSL_2) formal languages, which are languages where the words of the language can be distinguished from the ungrammatical by a tier-sequence of length 2. The model is provably capable of learning such languages, in the sense of Gold (1967), and, like D2L, iteratively removes segments from a tier that initially contains all segments. Jardine and McMullin (2017) extended the model to handle arbitrary values of k, and Lambert (2021) demonstrated that TSL languages are also learnable in an online setting. Jardine (2016b) applied Jardine and Heinz (2016)'s model to idealized natural language data, showing that the model successfully learns phonotactic restrictions in the setting where exceptions were removed and segments were preorganized into natural classes.

Of particular importance to the present paper is the model proposed by Burness and McMullin (2019) for learning Output Tier-Strictly 2-Local ($OTSL_2$) functions. While most prior work has dealt with phonotactic learning, Burness and McMullin's focus is on alternations. Moreover, Burness and McMullin's model shares some core characteristics with D2L. Burness and McMullin start with the definition of $OTSL_2$ functions and prove properties of that class of functions. These properties, together with the fact that output-strictly-local functions are a special case with all segments on the tier (section 2.4), lead to the development of a learning algorithm that starts with all segments on the tier and iteratively removes those that cannot be on the tier. Burness and McMullin (2021) improved the algorithm's time complexity.

By starting with a formal characterization of phonological phenomena and working out what properties an algorithm for learning such structures must have, formal-language-theoretic approaches attempt to solve a part of the learning problem from an analytical perspective, and are not necessarily intended for learning directly on naturalistic language data. The fact that D2L is derived from independent psychological mechanisms, is successful on naturalistic language data (section 5), and uses an analogous iterative change of representation, seems a desirable result for the overall approach.

3.3 Neural Network Models

There have been several attempts to model aspects of vowel harmony with recurrent neural networks (RNNs). Hare (1990) proposed using an RNN to model Hungarian vowel harmony, training it on synthetic bit sequences and finding that its assimilatory behavior on these bit sequences mirrored some of the complexities of Hungarian vowel harmony. Rodd

(1997), while not targeting vowel harmony directly, found that RNN models could use distributional information to learn phonological categories from a small Turkish corpus, and that they could learn to treat [+back] and [−back] vowels differently.

These early models were limited in their empirical scope, but in the decades since, research on neural networks (NNs) has accelerated. NNs have been used for morphological reinflection (e.g., Cotterell *et al.* 2016) and to revisit connectionism in the "past-tense debate" of English morphology (Kirov and Cotterell, 2018). While NNs have been questioned as cognitive models of morphophonological learning (e.g., McCurdy, Goldwater, and Lopez 2020; Belth *et al.* 2021) and these more recent models have not directly been applied to modeling long-distance alternations, some of the languages included in the SIGMORPHON reinflection task involve nonlocal dependencies. Moreover, Smith *et al.* (2021) used an RNN-based model to evaluate an articulatory account of height harmony in Nzebi by training the model on simulated speech to map segments to vocal-tract articulator movements.

Because prior works' problem settings have varied, it is difficult to draw conclusions about an RNN-based neural architecture's ability to model nonlocal alternations, but such an architecture is certainly applicable as a comparison model for D2L (see section 4.1.1 and the appendix for details on how I used it as such).

4 Comparison to Human Behavior

4.1 Model Behavior on Finley (2011)

I first compare to Finley (2011)'s artificial language experiment. Finley's experiment presented participants with training data consisting of <stem, suffixed> pairs. The suffix contained a sibilant that harmonized with a stem sibilant across an intervening vowel (36).

(36) /dis̱o-s̱u/ → [dis̱os̱u]

/nes̱i-s̱u/ → [nes̱is̱u]

/piʃ̠a-s̠u/ → [piʃ̠aʃ̠u]

/kuʃ̠o-s̠u/ → [kuʃ̠oʃ̠u]

To make the sibilants adjacent on a tier, the vowels must be excluded, suggesting [+cons] is the relevant tier. This predicts that when vowels and nonsibilant consonants intervene, the nonsibilant consonants will block the harmony. This prediction is borne out. After training, participants were evaluated in a two-alternative forced choice (2AFC) paradigm, where they were presented with a stem and two choices for its suffixed from: one harmonizing and one not. Learners generalized to novel instances like the training instances (37a), demonstrating that they learned a harmony pattern. However, they showed no preference for harmony in novel cases where nonsibilant consonants also intervened (37b), choosing the nonharmonizing option as often as a control group did.

(37) a. /bas̠o-s̠u/ → ✓ [bas̠os̠u] *[bas̠oʃ̠u]

/deʃ̠e-s̠u/ → ✓ [deʃ̠eʃ̠u] *[deʃ̠es̠u]

b. /ʃ̠eta-s̠u/ → ? [ʃ̠etaʃ̠u] ? [ʃ̠etas̠u]

/ʃ̠omi-s̠u/ → ? [ʃ̠omiʃ̠u] ? [ʃ̠omis̠u]

In contrast, a second experiment presented participants with training data where sibilants harmonized across both intervening vowels and nonsibilant consonants (38).

(38) /s̠uge-s̠u/ → [s̠uges̠u]

/s̠one-s̠u/ → [s̠ones̠u]

/ʃ̠upe-s̠u/ → [ʃ̠upeʃ̠u]

/ʃ̠ako-s̠u/ → [ʃ̠akoʃ̠u]

In this case, the [+sib] tier is needed in order for the sibilants to be adjacent, in which case learners should generalize to cases where only vowels intervene. The prediction is again borne out. Learners generalized to novel train-like instances (39a), and instance where only a vowel intervened (39b).

(39) a. /ʃ̱ika-s̱u/ → ✓ [ʃ̱ikaʃ̱u] *[ʃ̱ikas̱u]
/ʃ̱ege-s̱u/ → ✓ [ʃ̱egeʃ̱u] *[ʃ̱eges̱u]
b. /keʃ̱u-s̱u/ → ✓ [keʃ̱uʃ̱u] *[keʃ̱us̱u]
/niʃ̱a-s̱u/ → ✓ [niʃ̱aʃ̱u] *[niʃ̱as̱u]

4.1.1 Comparison Models

I compared D2L to a number of alternative models. GR is Goldsmith and Riggle (2012)'s phonotactic model (see also section 3 and section 6). To compare the relative wellformadness of candidates, I used the Boltzmann score from p. 882 of their paper. Following the authors, I used Laplace smoothing with a 0.5 smoothing-factor.

GG is Gouskova and Gallagher (2020)'s phonotactic model (see section 3 and section 6 for further description). I used the authors' code[8] and default parameters.

TSLIA is Jardine and Heinz (2016); Jardine and McMullin (2017)'s formal-language-theoretic model. I used the implementation from Aksënova (2020). This model is binary: it accepts or rejects a candidate string. When more than one candidate is accepted by the model, one candidate is chosen at random. I used TSLIA instead of Burness and McMullin (2019)'s model because I am not aware of an implementation of the latter.

LSTM is a Recurrent Neural Network (RNN) sequence-to-sequence model. The model is trained to predict the surface form of each underlyingly underspecified segment. This simplifies the learning problem by not requiring the model to predict the surface form for the entire sequence, but makes for a fair comparison to D2L, which also has access to the underlying forms. I used a Pytorch (Paszke *et al.*, 2019) implementation, and discuss architecture specifics, training procedure, and hyperparameter tuning in the appendix.

3G is a trigram phonotactic model, which assigns a probability to each candidate in terms of the trigrams it contains—how frequent they are in the training data. As in GR, I used Laplace smoothing with a smoothing-factor of 0.5.

4.1.2 Setup

I use the training and test items reported in Finley 2011:15's appendix. Each model is trained on the training instances, then probed to choose between each pair of items in the 2AFC test set. I treat the sibilant in [-su]/[-ʃu] as underlyingly /S/, unspecified for [ant]. The comparison phonotactic models are trained on the surface forms, and, for each test trial, the model assigns a score to the [-su] and [-ʃu] forms and the form with the higher score is chosen. If the model assigns the same score to both items, then the choice is made at random. Since D2L produces an output for an input, I use its produced form as its choice if the produced form matches one of the 2AFC choices. Otherwise the choice is made at random. To simulate multiple participants, I run each model 30 times and report averages and standard deviations. This is important because some randomness is introduced due to GR, GG, and 3G being stochastic, and because when a model assigns the same score to both 2AFC choices, the choice is made at random. Following Finley (2011), each of the 24 training items appears 5 times in the total exposure set, and the items are presented in a random order for each of the 30 runs. I list the segment features in the appendix.

4.1.3 Results

The results for the first experiment are given in table 1, and for the second experiment in table 2. The tables report, for each model, the fraction of the test instances where the model picked the harmonizing choice. The Hum row records a "✓" whenever Finley (2011) reported that the experimental-group participants chose the harmonizing choice significantly more often than the control-group participants. It records an "✗" wherever the two groups chose

Table 1: Results for Finley (2011)'s first experiment, where training instances involved sibilants harmonizing across intervening vowels. Test instances are of three types: train (Old), novel train-like (New Train-Like), and novel items where both vowels and nonsibilant consonants intervene between sibilants (Novel). D2L generalizes in exactly the cases where humans do, and does not generalize in exactly the cases where humans do not.

	Train CV$\underline{\text{S}}$V-$\underline{\text{S}}$V		
	CV$\underline{\text{S}}$V-$\underline{\text{S}}$V (Old)	CV$\underline{\text{S}}$V-$\underline{\text{S}}$V (New Train-Like)	$\underline{\text{S}}$VCV-$\underline{\text{S}}$V (Novel)
Hum	✓	✓	✗
D2L	1.0000 ± 0.00	1.0000 ± 0.00	0.3333 ± 0.00
GG	0.5389 ± 0.13	0.5394 ± 0.16	0.4500 ± 0.14
GR	0.4667 ± 0.27	0.6364 ± 0.18	0.5333 ± 0.07
LSTM	0.8917 ± 0.17	0.7970 ± 0.18	0.8611 ± 0.25
TSLIA	0.5389 ± 0.13	0.5394 ± 0.16	0.4500 ± 0.14
3G	1.0000 ± 0.00	1.0000 ± 0.00	0.7056 ± 0.04

the harmonizing choice at statistically indistinguishable rates.[9] If, over a set of test items, a model makes the harmonizing choice significantly more often than a control model that makes a random selection from the two choices, then I treat the model as having generalized to those test items. The test of significance is made with a one-sided t-test that compares the average model performance over 30 runs to that of the random control model. The null hypothesis is that the tested model's average performance is equal to the random control model's. I use Welch's t-test, which does not assume equal variance, and a significance level of $\alpha = 0.99$. I shade a cell gray if the model matches the human result—that is iff either both Hum and the model generalized or neither generalized.

Table 2: Results for Finley (2011)'s second experiment, where training instances involved sibilants harmonizing across both vowels and nonsibilant consonants. Test items are of three types: train (Old), novel train-like (New Train-Like) and novel items where only vowels intervene between sibilants (Novel). Both humans and D2L learned a harmony pattern (Old) and extend it to both New Train-Like and Novel test instances.

	Train $\underline{\text{S}}$VCV-$\underline{\text{S}}$V		
	$\underline{\text{S}}$VCV-$\underline{\text{S}}$V (Old)	$\underline{\text{S}}$VCV-$\underline{\text{S}}$V (New Train-Like)	CV$\underline{\text{S}}$V-$\underline{\text{S}}$V (Novel)
HUM	✓	✓	✓
D2L	1.0000 ± 0.00	1.0000 ± 0.00	1.0000 ± 0.00
GG	0.4944 ± 0.16	0.4778 ± 0.16	0.4667 ± 0.14
GR	1.0000 ± 0.00	0.4167 ± 0.00	0.2500 ± 0.00
LSTM	0.9167 ± 0.19	0.8889 ± 0.30	0.7139 ± 0.32
TSLIA	0.5444 ± 0.15	0.4944 ± 0.16	0.4500 ± 0.14
3G	1.0000 ± 0.00	0.5833 ± 0.00	0.2500 ± 0.00

4.1.4 Discussion

D2L matches the human results in all cases. When trained on CV$\underline{\text{S}}$V-$\underline{\text{S}}$V words, D2L learns a [+cons] tier (40a), and thus produces harmony in novel CV$\underline{\text{S}}$V-$\underline{\text{S}}$V (second column, table 1), but not $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words (third column, table 1). In contrast, when trained on $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words, D2L learns a [+stri] tier (40b), which produces harmony in both novel $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words (second column, table 2) and novel CV$\underline{\text{S}}$V-$\underline{\text{S}}$V words (third column, table 2).

(40) a. Agree({S}, {ant})/[+cons] __ $\circ$ proj($\cdot$, [+cons])

b. Agree({S}, {ant})/[+stri] __ $\circ$ proj($\cdot$, [+stri])

D2L's performance is categorical because, as an algorithm, it abstracts away from experimental complexities, like the fact that human participants may lose attention or not understand the task.

The comparison models are largely ineffective at generalizing at all from the limited training data, with the exception of LSTM. However, when trained on CV$\underline{\text{S}}$V-$\underline{\text{S}}$V words, LSTM generalizes to both CV$\underline{\text{S}}$V-$\underline{\text{S}}$V and $\underline{\text{S}}$VCV-$\underline{\text{S}}$V test words, whereas humans do not generalize to $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words (table 1). Thus, LSTM fails to exhibit the blocking behavior of nonsibilant consonants in the first experiment.

GR is able to achieve moderate performance generalizing to CV$\underline{\text{S}}$V-$\underline{\text{S}}$V words when trained on words of the same type (second column, table 1), and for $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words, the intervening C prevents the sibilants from being adjacent on the consonant tier that the Hidden Markov Model learns (third column, table 1). However, when trained on $\underline{\text{S}}$VCV-$\underline{\text{S}}$V, no interactions between sibilants are visible to the model and it is unable to generalize beyond the training data (table 2).

GG appears unable to generalize from these small datasets. While this may seem to be a limitation of the data, note that humans were able to generalize from the same training data. In the next section, we turn to McMullin and Hansson (2019)'s study, which involves substantially more data, and is thus more instructive regarding GG's generalization behavior.

TSLIA's performance indicates that the data does not contain the model's characteristic sample. 3G frequently chooses the harmonizing form for $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words when trained on CV$\underline{\text{S}}$V-$\underline{\text{S}}$V words, and rarely chooses the harmonizing form for CV$\underline{\text{S}}$V-$\underline{\text{S}}$V words when trained on $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words. This is the opposite of what humans did.

4.2 Model Behavior on McMullin and Hansson (2019)

McMullin and Hansson (2019) replicated results similar to Finley (2011)'s, extending them to both liquid harmony and dissimilation. In these experiments the harmony/dissimilation was *regressive*. Participants were first exposed to a practice phase where they were presented with verb stems followed by a past-tense form, which added a [-ɹu] suffix to the stem, and the same stems followed by a future-tense form, which added a [-li] suffix to the stem. These practice-phase stems did not contain liquids: they allowed the participants to learn the relevant morphology.

Next, in the training phase, participants were presented with <stem, past, future> triplets. The two training settings of Finley 2011 where doubled by McMullin and Hansson (2019), performing each experiment in both assimilatory and dissimilatory settings. In all experiments, the data contained 50% distractors, which were stems with no liquid.

In what the authors labeled experiment 1a, the stem liquid harmonized regressively with the affix liquid across an intervening vowel (41) (treating the alternating stem-liquid as underlyingly underspecified /L/).

(41) /toboL̲e-ɹ̲u/ → [toboɹ̲eɹ̲u]
/toboL̲e-l̲i/ → [tobol̲el̲i]
/dumiL̲i-ɹ̲u/ → [dumiɹ̲iɹ̲u]
/dumiL̲i-l̲i/ → [dumil̲il̲i]

Symmetrically, in experiment 2a, the stem liquids *dissimilated* regressively from the affix liquid across an intervening vowel (42).

(42) /toboL̲e-ɹ̲u/ → [tobol̲eɹ̲u]
/toboL̲e-l̲i/ → [toboɹ̲el̲i]
/dumiL̲i-ɹ̲u/ → [dumil̲iɹ̲u]
/dumiL̲i-l̲i/ → [dumiɹ̲il̲i]

Like Finley (2011)'s study, participants generalized the training pattern to novel words of the same CVCVL̲V-L̲V form, but did not extend it to CVL̲VCV-L̲V or L̲VCVCV-L̲V forms, where the liquids crossed more than just vowels. This is the predicted behavior if participants construct a [+cons] tier.

In experiments 1b and 2b, the participants were presented with assimilatory (43a) and dissimilatory (43b) instances of the form CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V.

(43) a. /te$\underline{\text{L}}$omu-$\underline{\text{ɹ}}$u/ → [te$\underline{\text{ɹ}}$omu$\underline{\text{ɹ}}$u]
/te$\underline{\text{L}}$omu-$\underline{\text{l}}$i/ → [te$\underline{\text{l}}$omu$\underline{\text{l}}$i]
/po$\underline{\text{L}}$eku-$\underline{\text{ɹ}}$u/ → [po$\underline{\text{ɹ}}$eku$\underline{\text{ɹ}}$u]
/po$\underline{\text{L}}$eku-$\underline{\text{l}}$i/ → [po$\underline{\text{l}}$eku$\underline{\text{l}}$i]

b. /te$\underline{\text{L}}$omu-$\underline{\text{ɹ}}$u/ → [te$\underline{\text{l}}$omu$\underline{\text{ɹ}}$u]
/te$\underline{\text{L}}$omu-$\underline{\text{l}}$i/ → [te$\underline{\text{ɹ}}$omu$\underline{\text{l}}$i]
/po$\underline{\text{L}}$eku-$\underline{\text{ɹ}}$u/ → [po$\underline{\text{l}}$eku$\underline{\text{ɹ}}$u]
/po$\underline{\text{L}}$eku-$\underline{\text{l}}$i/ → [po$\underline{\text{ɹ}}$eku$\underline{\text{l}}$i]

A liquid tier is needed for the training liquids to be adjacent, predicting that learners will extend the pattern to CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V and $\underline{\text{L}}$VCVCV-$\underline{\text{L}}$V forms. These predictions were borne out.

4.2.1 Setup

I follow McMullin and Hansson (2019:sec. 2)'s setup to produce the stimuli. The setup details are the same as in the prior experiment (section 4.1.2) except that, following McMullin and Hansson (2019), the stimuli are only presented once each. The stimuli include the practice phase forms. I treat alternating stem liquids [l]/[ɹ] as underlyingly underspecified /L/. I use the same comparison models as in the prior experiment (section 4.1.1) and list the segment features in the appendix.

4.2.2 Results

The results for the experiments with CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V exposure data are given in table 3, and the results for the experiments with CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V exposure data in table 4. For the assimilation experiments, the tables report, for each model, the fraction of the test instances where the

Table 3: Results from McMullin and Hansson (2019)'s Experiment 1a (assimilation) and 2a (dissimilation), where training instances involved liquids interacting across intervening vowels. D2L matches human behavior in all cases.

	Train CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V (assimilation)			Train CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V (dissimilation)		
	CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V	CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V	$\underline{\text{L}}$VCVCV-$\underline{\text{L}}$V	CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V	CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V	$\underline{\text{L}}$VCVCV-$\underline{\text{L}}$V
Hum	✓	✗	✗	✓	✗	✗
D2L	1.000 ± 0.00	0.394 ± 0.07	0.471 ± 0.05	1.000 ± 0.00	0.419 ± 0.07	0.467 ± 0.05
GG	0.477 ± 0.10	0.617 ± 0.08	0.493 ± 0.06	1.000 ± 0.00	0.546 ± 0.04	0.475 ± 0.02
GR	0.972 ± 0.08	0.652 ± 0.07	0.495 ± 0.05	0.996 ± 0.02	0.368 ± 0.06	0.510 ± 0.05
LSTM	0.850 ± 0.23	0.840 ± 0.22	0.795 ± 0.22	0.833 ± 0.24	0.831 ± 0.23	0.820 ± 0.23
TSLIA	0.503 ± 0.09	0.520 ± 0.09	0.489 ± 0.07	0.497 ± 0.09	0.480 ± 0.09	0.511 ± 0.07
3G	0.969 ± 0.00	0.637 ± 0.06	0.505 ± 0.09	1.000 ± 0.00	0.362 ± 0.06	0.495 ± 0.09

model picked the harmonizing choice. For the dissimilatory experiments, they report the fraction where the disharmonizing choice was made. The Hum row records a "✓" whenever McMullin and Hansson (2019) reported that the experimental-group participants extended the training pattern to test instances of the form in the corresponding column, and an "✗" where they did not. Gray cells mark where the models match the human result. As before, the models are compared to a control model that makes a random selection in the 2AFC test.

4.2.3 Discussion

D2L matches the human results in every setting, and is the only model to do so. When trained on CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V words, D2L learns (44a) and (44c), with a [+cons] tier projection. When trained on CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V words, D2L learns (44b) and (44d), with a {l, ɹ, L} liquid

Table 4: Results from McMullin and Hansson (2019)'s. Experiment 1b (assimilation) and 2b (dissimilation), where training instances involved liquids interacting across intervening vowels. D2L matches human behavior in all cases.

	Train CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V (assimilation)			Train CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V (dissimilation)		
	CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V	CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V	$\underline{\text{L}}$VCVCV-$\underline{\text{L}}$V	CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V	CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V	$\underline{\text{L}}$VCVCV-$\underline{\text{L}}$V
Hum	✓	✓	✓	✓	✓	✓
D2L	1.000 ± 0.00	1.000 ± 0.00	1.000 ± 0.00	1.000 ± 0.00	1.000 ± 0.00	1.000 ± 0.00
GG	0.512 ± 0.04	0.497 ± 0.04	0.495 ± 0.04	1.000 ± 0.00	1.000 ± 0.00	1.000 ± 0.00
GR	0.455 ± 0.09	0.641 ± 0.08	0.503 ± 0.06	0.560 ± 0.06	0.359 ± 0.08	0.499 ± 0.05
LSTM	0.767 ± 0.25	0.767 ± 0.25	0.767 ± 0.25	0.800 ± 0.24	0.800 ± 0.24	0.800 ± 0.24
TSLIA	0.503 ± 0.09	0.520 ± 0.09	0.489 ± 0.07	0.497 ± 0.09	0.480 ± 0.09	0.511 ± 0.07
3G	0.440 ± 0.06	0.641 ± 0.08	0.502 ± 0.10	0.560 ± 0.06	0.359 ± 0.08	0.498 ± 0.10

tier. These results also demonstrate that D2L is able to construct regressive rules and infer whether a process is assimilatory or dissimilatory (see section 2.3.4).

(44) a. $\text{Agree}(\{\text{L}\}, \{\text{ant}, \text{cor}, \text{lat}\}) /\ \_\_\ [+\text{cons}] \circ \text{proj}(\cdot, [+\text{cons}])$

b. $\text{Agree}(\{\text{L}\}, \{\text{ant}, \text{cor}, \text{lat}\}) /\ \_\_\ \{\text{l}, \text{ɹ}\} \circ \text{proj}(\cdot, \{\text{l}, \text{ɹ}, \text{L}\})$

c. $\text{Disagree}(\{\text{L}\}, \{\text{ant}, \text{cor}, \text{lat}\}) /\ \_\_\ [+\text{cons}] \circ \text{proj}(\cdot, [+\text{cons}])$

d. $\text{Disagree}(\{\text{L}\}, \{\text{ant}, \text{cor}, \text{lat}\}) /\ \_\_\ \{\text{l}, \text{ɹ}\} \circ \text{proj}(\cdot, \{\text{l}, \text{ɹ}, \text{L}\})$

When trained on CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V words (table 3), D2L is the only model to match human behavior in all cases, though some comparison models also come close. However, when trained on CV$\underline{\text{L}}$VCV-$\underline{\text{L}}$V words (table 4), only D2L and LSTM come close to human behavior. Similarly to the previous experiment (section 4.1), when trained on CVCV$\underline{\text{L}}$V-$\underline{\text{L}}$V words, LSTM generalizes to words where nonliquids intervened and humans did not generalize (table 3). These results suggest that LSTM may not be able to express blockers, instead learning a dependence between L's independent of what segments intervene.

Because the HMM of GR usually learns the consonant/vowel tiers, the interacting liquids in CVLVCV-LV words are blocked by the intervening C, which prevents the model from substantially generalizing. Similarly, the interacting liquids are beyond the trigram sensitivity of 3G, so its occasional above-chance performance is only due to coincidental statistical regularities in the exposure data unrelated to the liquid interaction. TSLIA's performance again indicates that the data does not contain the model's characteristic sample.

The asymmetry of GG between assimilation and dissimilation deserves some discussion. Because no -LVL- sequences occur in CVLVCV-LV training instances, the Hayes and Wilson 2008 model extracts a trigram constraint *[l,ɹ][][l,ɹ], which GG uses to project a liquid tier. In the assimilation experiment, GG learns the single constraint *[l,ɹ][l,ɹ] on this tier, not two constraints *[lɹ] and *[ɹl]. Consequently, it cannot distinguish between assimilitory and nonassimilatory sequences. In the dissimilation experiment, GG learns the single tier-constraint *[ll], as well as a nontier constraint *[−cor], which applies to [ɹ] but not [l]. Consequently [ɹl] » [ll] due to the tier constraint, which has a much higher weight than the nontier constraint. Moreover, [lɹ] » [ɹɹ] because the latter violates *[−cor] twice. Thus, the two constraints coincidentally conspire to yield a functional generalization. However, the coincidental result only works for dissimilation, and the asymmetry with assimilation is not consistent with human behavior, which was similar for assimilation and dissimilation.

5 Learning Natural Language Alternations

5.1 Turkish Vowel Harmony

The Turkish vowel inventory, (45), has 4 [+back] and 4 [−back] vowels, all of which participate in [±back] harmony (Kabak, 2011:2).

(45)

	front		back	
	unround	round	unround	round
high	i	y	ɯ	u
low	e	ø	ɑ	o

Affix vowels alternate between [+back] vowels when the final stem vowel is [+back] and [−back] when the final stem vowel is [−back], as shown in (46) repeated from (2).

(46) [dɑ̲l-lɑ̲r-ɯ̲n] [je̲r-le̲r-i̲n] [i̲p-le̲r-i̲n]
‘branch’-Pl-Gen ‘place’-Pl-Gen ‘rope’-Pl-Gen

In addition to back/front harmony, the [+high] vowels participate in secondary rounding harmony, as exemplified by the Gen affix (47). The [+high] vowels harmonize with the final vowel of the stem, both when the the stem vowel is [+high] (47a) and [−high] (47b). Examples from Nevins 2010:29; Kabak 2011:3.

(47)
a. [i̲p-i̲n]
‘rope’-Gen
[jy̲z-y̲n]
‘face’-Gen
[kɯ̲z-ɯ̲n]
‘girl’-Gen
[bu̲z-u̲n]
‘ice’-Gen

b. [e̲l-i̲n]
‘hand’-Gen
[sø̲z-y̲n]
‘word’-Gen
[sɑ̲p-ɯ̲n]
‘stalk’-Gen
[jo̲l-u̲n]
‘road’-Gen

I follow Nevins (2010) and Belth (2023) in treating primary and secondary harmony as a single process, where alternating low vowels are underspecified for their [back] value underlyingly, and alternating [+high] vowels are underspecified for both [back] and [round]. Consequently, Agree is taken to copy only the underspecified vocalic features from the closest vowel—that is, both [back] and [round] for [?back, +high, ?round], but only [back] for

[?back, ±high, ±round]. The fact that alternating [+high] vowels take their [round] value from *any* vowel, regardless of height, is evidence in favor of this treatment. If we were to treat primary and secondary harmony as separate processes, D2L could be run twice—once for feature [±back] and once for [±round]—to construct two generalizations.

Some affixes do not participate in vowel harmony, and some are *half-harmonizing*, meaning that one of two vowels participates. These exceptions do not alternate on the surface, so there is no motivation for underspecification (Nevins, 2010:sec. 2.6).

Some Turkish suffixes, such as the locative, also exhibit a local voicing assimilation process, where an affix-initial alveolar stop matches the voicing of the preceding segment (48; examples from Dobrovolsky 1982; Çöltekin 2010; Kornfilt 2010).

(48) [byro-dɑ]
'office'-Loc
[ev-de]
'house'-Loc
[ʤep-te]
'pocket'-Loc

5.1.1 Setup

I use two datasets, which I created in Belth 2023 from CHILDES (MacWhinney, 2014) and MorphoChallenge (Kurimo *et al.*, 2010) respectively. Both datasets contain frequency-annotated, IPA-transcribed surface forms. The CHILDES dataset contains 1,727 word types and the MorphoChallenge dataset contains 22,315. I used morphological analyses available in the datasets to construct underlying forms. I set the underlying form of each segment in each morpheme to match its surface form unless the segment appears in different forms across realizations of the morpheme. For alternating segments, I set any feature that alternated on the surface as underlyingly unspecified. For instance, the vowel of the Pl affix

[-lɑr]/[-ler] is set to /?back, −high, −round/, and the GEN affix vowel [-in]/[-yn]/[-ɯn]/[-un] is set to /?back, +high, ?round/. This process results in underspecified URs /A/ with extension {ɑ, e}, /H/ with extension {i, y, ɯ, u}, and /D/ with extension {d, t}. I list the segment features in the appendix.

For training data, I sampled 1K unique words, weighted by frequency, and used the remaining of words for testing (727 for CHILDES and 21,315 for MORPHOCHALLENGE). I repeated this 30 times with a different random sample each time. I report the average performance across the 30 runs. The comparison phonotactic models are trained on the surface forms, and each test surface form is predicted by computing all possible specifications of the input UR's underspecified segments and choosing the form that the model assigns the highest score to. For instance, for input /dɑl-lAr-Hn/, the candidate surface forms are [dɑl-lɑr-in], [dɑl-lɑr-yn], [dɑl-lɑr-ɯn], [dɑl-lɑr-un], [dɑl-ler-in], [dɑl-ler-yn], [dɑl-ler-ɯn], [dɑl-ler-un]. The comparison models are the same as the prior experiments.

Since the data involves both vowel harmony and local voicing assimilation, D2L seeks to create a generalization for each process. This is accomplished automatically (i.e., not hard-coded) by following Belth (To appear) in running the search for a generalization whenever a new discrepancy is found (e.g., /A/ → [e] or /D/ → [t]).

5.1.2 Results

The results shown in table 5 demonstrate that D2L learns generalizations that robustly predict the surface form of unseen test words. D2L's accuracy is higher than the comparison models and is consistent with acquisition studies, which reveal that Turkish-speaking children as young as 2;0—when their vocabulary likely contains no more than a thousand words—already know vowel harmony well enough to extend it to nonce words (Altan, 2009). D2L's errors are due to genuine exceptions, which it is able to tolerate and still find a generalization by lexicalizing these under the Tolerance Principle. For example, D2L

Table 5: Accuracy of models on held-out test words, when learning Turkish vowel harmony.

	Test Accuracy	
Model	CHILDES	MorphoChallenge
D2L	**0.9955 ± 0.00**	**0.9872 ± 0.00**
GG	0.9628 ± 0.02	0.9083 ± 0.05
GR	0.9810 ± 0.01	0.9291 ± 0.08
LSTM	0.8866 ± 0.07	0.8290 ± 0.13
TSLIA	0.6421 ± 0.01	0.6388 ± 0.00
3G	0.8213 ± 0.01	0.7421 ± 0.01

predicts *[saatlɑr] for the word [saat-ler] 'watch'-Pl, which originates from Arabic and exceptionally does not harmonize in adult speech. Like D2L, children appear to overextend vowel harmony to exceptional words (Altan, 2009). The rules D2L constructed are (49).

(49) a. $\text{Agree}([?\text{back}], \{\text{back}, \text{round}\}) / [-\text{cons}] \_\_ \circ \text{proj}(\cdot, [-\text{cons}])$

b. $\text{Agree}([?\text{voice}], \{\text{voice}\}) / [*] \_\_$

The vowel harmony rule (49a) states that vowels with unspecified backness (i.e., [?back]) take their [back] value and, if also unspecified [?round], their [round] value from a vowel to the left on a projected vowel tier. The voicing assimilation rule (49b) matches /D/'s voicing to any segment (denoted by [*]) to its left. I omit the projection component from this rule since it trivially maps all segments, which demonstrates the D2L does not change representations when string-adjacent dependencies can predict the alternation (section 2.4). The generalizations are consistent with standard descriptive analyses (Kabak, 2011).

5.2 *Finnish Vowel Harmony*

The Finnish vowel inventory (50) (Suomi, Toivanen, and Ylitalo 2008:sec. 3.1) has 8 vowels, 6 of which participate in front/back harmony, as discussed in section 1 and repeated in (51).

		front		back	
		unround	round	unround	round
(50)	high	i	y		u
	mid	e	ø		o
	low	æ		ɑ	

The vowels {i, e} are neutral, neither participating in nor blocking harmony (51b). When a stem contains only neutral vowels, alternating affix vowels are [−back] by default (51c). Examples from Ringen and Heinämäki 1999:304-305; Nevins 2010:76.

(51) a. [pøytæ-næ] 'table'-Ess; [pouta-na] 'fine weather'-Ess

b. [koti-nɑ] 'home'-Ess

c. [velje-næ] 'road'-Ess

Since the neutral vowels are [−back], the default case (51c) could instead be characterized as the affix vowel harmonizing with the neutral vowels only when no non-neutral vowels are present. However, the analysis with a default value is preferred, because it extends to Uyghur, which has the same vowel organization, but alternating vowels are [+back] when the stem contains only [−back] neutral vowels (Lindblad 1990; Nevins 2010:77-78).

5.2.1 *Setup*

Finnish data comes from the MorphoChallenge (Kurimo *et al.*, 2010), which includes 1835 frequency-annotated words with morphological segmentations. The surface form of these affixes varied extensively in ways beyond just vowel alternations (e.g., the GEN affix has forms [n], [en], [ten], ...) making it challenging to compute precisely how vowels alternate. Consequently, I treated all affix vowels as alternating and underlyingly unspecified for feature [back]. This choice potentially overestimates the number of harmony exceptions, and thus potentially makes the problem more challenging for D2L, while not affecting the comparison phonotactic models, which learn directly from surface forms. I dropped words with only one occurrence, yielding a set of 1219 words. Finnish orthographic vowels map directly to phonemes. I mapped orthographic consonants to phonemes directly, but treated <x> as [ks] and <c>, <q> as [k]. The segment features are in the appendix.

For training data, I sampled 80% of words (975) weighted by frequency and used the other 20% for testing. I repeated this sampling 30 times, and report the average performance across the 30 runs. To to get a predicted surface form for the phonotoactic comparison models, I again computed candidate surface forms by permuting affix vowels between back and front, and selecting the candidate that the model assigns the highest score to.

5.2.2 Results

The accuracies shown in table 6 reveal that D2L is the most accurate model. D2L learns the generalization (52), where unspecified vowels take their [±back] feature from the vowel to the left, skipping consonants and neutral vowels (52a). D2L also learned that if no non-neutral vowel is to the left, the surface vowel should be [−back] (52b).

(52) a. $\text{Agree}([\text{?back}], \{\text{back}\}) / [-\text{cons}] \_\_ \circ \text{proj}(\cdot, [-\text{cons}] \setminus \{\text{i}, \text{e}\})$

b. Elsewhere [−back]

The tier correctly excludes the neutral vowels while preserving all other vowels. On a small number of simulations, the tier contained one or two spurious segments—[f] and/or [b]—because they never occur between an affix vowel and a stem vowel in the training

Table 6: Accuracy of models on held-out test words, when learning Finnish vowel harmony.

Model	Test Accuracy
D2L	**0.9799** ± **0.01**
GG	0.8113 ± 0.03
GR	0.8441 ± 0.03
LSTM	0.8070 ± 0.06
TSLIA	0.8350 ± 0.02
3G	0.8418 ± 0.02

data and, consequently, never interfere with harmony. The elsewhere condition matches the default value given in accounts of Finnish for what happens when a stem contains only neutral vowels (Ringen and Heinämäki, 1999). Note that because the segments do not fall neatly into a natural class, D2L simply listed the tier segments explicitly; I presented it here with natural classes and set operations for clarity.

5.3 Latin Liquid Dissimilation

In the Latin adjectival *-alis*/*-aris* affix, default /l/ (5a) dissimilates to [r] when preceded by /l/ across varying distances, as shown in (5b) and repeated in (53b). The dissimilation of /l/ is blocked by intervening /r/ (53c). Cser (2010) argues that intervening [−cor] consonants also block dissimilation (53d).

(53)	a. *nav-alis*	b. *popul-aris*	c. *flor-alis*	d. *pluvi-alis*
	‘naval’	‘popular’	‘floral’	‘rainy’
		lun-aris		*leg-alis*
		‘lunar’		‘legal’

Table 7: Accuracy of models on held-out test words after learning Latin liquid dissimilation.

Model	Test Accuracy
D2L	**0.9708 ± 0.03**
GG	0.2708 ± 0.07
GR	0.8861 ± 0.12
LSTM	0.7444 ± 0.09
TSLIA	0.2708 ± 0.07
3G	0.9194 ± 0.04

5.3.1 Setup

The data comes from the Perseus project (Smith, Rydberg-Cox, and Crane, 2000), which contains Old and Classical Latin texts from the 3rd century BCE through the 2nd century CE. Words are annotated for frequency. I used words containing two liquids and ending in *-alis*/*-aris*, removing nonadjectives. The result is a dataset of 121 words. I treat /-aLis/ as the underlying form of [-*alis*]/[-*aris*], where /L/ is unspecified for [?lat]. I map orthographic segments directly to phonemes, treating <v> as the semivowel [w], <c> as [k], <x> as [ks], and <ll> as [l]. I also dropped the <h> from <th>, <kh>, and <ph>, treating <h> as marking aspiration. I list the segment features in the appendix.

The training data is an 80% frequency-weighted sample of words (97), and the testing data is the remaining 20% of words. This sampling was repeated 30 times, and models' performances are computed as averages over these 30 train/test splits.

5.3.2 Results

Again (table 7), D2L is the most accurate model. D2L discovered rule (54), where /L/ takes the opposite value of [lat] from the consonant to its left on a consonant tier.

(54) $\mathrm{Disagree}([?\mathrm{lat}], \{\mathrm{lat}\})/[+\mathrm{cons}]\ \_\_ \circ \mathrm{proj}(\cdot, [+\mathrm{cons}])$

I used features from Cser (2010), in which [l] is the only [+lat] segment. Thus, this rule in fact says that /L/ disharmonizes to [+lat] [l] when tier-adjacent to any consonant except [+lat] [l], for which it disharmonizes to [−lat] [r]. In effect then, [l] surfaces for /L/, except when tier-adjacent to a preceding [l], in which case it surfaces as [r]. Thus [r] and the [−cor] consonants, which block dissimilation according to Cser (2010), are preserved on the tier, consistent with Cser's analysis. The [+cor] consonants {s, t, d, n} are also included on the tier. Cser (2010) notes that there is no data either way for whether [d] blocks. In my data, there is insufficient motivation for D2L to remove {s, t, n} from the tier.

6 Discussion

I intend D2L as a proposal for how humans might learn phonological alternations, at roughly the *algorithmic level* in my interpretation of the sense of Marr (1982). That is, D2L constitutes a hypothesis that human learners roughly follow the steps laid out by D2L—tracking adjacent dependencies and iteratively discarding those that prevent prediction of the surface form. This paper constitutes a presentation of that hypothesis (section 2) and independent motivation for it (section 2.1), followed by initial supporting evidence on the grounds of prior artificial language studies (section 4) and new computational modeling experiments (section 5). As an explicit computational model, it can be used to generate predictions for further experimental studies.

D2L seems of importance to phonological theory because it demonstrates that phonological tiers can be learned from naturalistic data, even in the presence of exceptions, by tracking only adjacent dependencies, and that such representations arise naturally from a learner grounded in humans' proclivity for tracking adjacent dependencies (section 2.1). Moreover, if D2L is on the right track regarding how humans generalize, then it may hold some explanatory power for why phonological structures are the way they are (Dresher, 1999; Heinz, 2009, 2010). In particular, because D2L is restricted to only tracking adjacent dependencies, the only option it has when it is unable to predict the surface form of an alternating segment is to delete the adjacent segments that are getting in the way and hope that the relevant dependencies will then be accessible adjacently on the new representation. Thus, the fact that a wide range of phonological phenomena can be described in terms of adjacent dependencies on some tier (Heinz, 2010; Jardine and Heinz, 2016; Burness and McMullin, 2021) could be a reflection of the fact that the human mind's attention is initially drawn to adjacent items, and the natural resulting learning algorithm produces tier-like representations as a byproduct.

6.1 D2L Compared to Other Models

I will highlight how D2L compares to other models, and why its functioning leads it to match human behavior in the artificial language experiments (section 4) and succeed at learning natural language alternations (section 5).

The artificial language experiments (section 4) used poverty-of-stimulus paradigms, which precisely design the exposure (training) data so as to underdetermine what process underlies the observed alternation. Thus, when Finley (2011)'s exposure data contains CV$\underline{\text{S}}$V-$\underline{\text{S}}$V words, it could be that harmony applies strictly to sibilants across intervening vowels (i.e. $\underline{\text{S}}$V$\underline{\text{S}}$) or also across intervening nonsibilants (e.g. $\underline{\text{S}}$VCV$\underline{\text{S}}$V). The exposure data contains no words where sibilants are separated by more than a vowel, so the exposure data underdetermines which of these generalizations underlies the harmony. Similarly, when the exposure data contains $\underline{\text{S}}$VCV-$\underline{\text{S}}$V words, it could be that harmony applies strictly to sibilants where both consonants and vowels intervene (but not when *only* a vowel intervenes) or whenever anything other than a sibilant intervenes. Again, the choice is underdetermined. The same logic holds for McMullin and Hansson (2019)'s experiments.

As evidenced by the human behavior, something is critically different between the cases where the exposure data contains CV$\underline{S}$V-$\underline{S}$V words compared to when it contains $\underline{S}$VCV-$\underline{S}$V words. D2L's incremental deleting of adjacent dependencies leads it to delete only vowels (V) when exposed to CV$\underline{S}$V-$\underline{S}$V words, and both vowels and nonsibilant consonants (V and C) when exposed $\underline{S}$VCV-$\underline{S}$V words, mirroring the asymmetry observed in human behavior. In contrast, consider the way GG constructs a tier. When the exposure data is CV$\underline{S}$V-$\underline{S}$V words and $\underline{S}$VCV-$\underline{S}$V words, the Hayes and Wilson 2008 model, which GG uses, may observe that nonharmonizing $\underline{S}$V$\underline{S}$ sequences are conspicuously absent and learn the constraints *[s][][ʃ] and *[ʃ][][s]. When the exposure data is $\underline{S}$VCV-$\underline{S}$V words, it may notice that *any* (harmonizing or not) $\underline{S}$V$\underline{S}$ sequences are conspicuously absent and learn the constraints *[s][][ʃ], *[s][][s], *[ʃ][][s], and *[ʃ][][ʃ].[10] In both cases, because GG uses the smallest natural class containing X and Y in *X[]Y constraints, it will project the smallest natural class containing {s, ʃ}, which is the [+stri] tier. Thus, fundamentally, GG is not capable of distinguishing the case where sibilants harmonize only across intervening vowels from the case where sibilants harmonize across both vowels and nonsibilant consonants.

We see the same situation when we turn to natural language data, where D2L is again able to handle blockers in Latin liquid dissimilation (section 5.3), but GG, which may induce the constraint *[l][][l] if such sequences are sufficiently absent in the training data, induces the smallest natural class containing {l}, which would be [+lat], since [l] is the only lateral consonant. As a result, neither the [r] nor [−cor] blockers will be projected on the tier. In other words, GG is only sensitive to the harmonizing or dissimilating segments and thus cannot capture blockers unless they coincidentally fall in the smallest natural class subsuming X and Y in the *X[]Y constraints that Hayes and Wilson (2008)'s model extracts.

Goldsmith and Riggle (2012)'s model (GR), which was intended as an information-theoretic attempt to justify the use of a vowel tier in analyzing Finnish vowel harmony, used Goldsmith and Xanthos (2009)'s HMM to extract tiers. However, this approach is only well-suited for extracting the consonant and vowel tiers. This is because Goldsmith and Xanthos

(2009)'s HMM is a state-transition model with two hidden states; it extracts the two classes of segments that maximizes the probability of the data. Intuitively, this is maximized when the two hidden categories transition sequentially in words. In such a case, the probability of transitioning from one state to the next will be high and the probability of staying in the current state will be low. In most imaginable linguistic cases, words tend to switch between consonants and vowels—for example CVCVCV and CVCCVC are more frequent than CCVV or CCCCVCC. This is hardly a categorical fact, but it is a robust statistical trend. Thus, the HMM's two hidden states tend to robustly correspond to the categories *consonant* and *vowel*. For this reason, GR is not able to flexibly extend to alternations involving tiers other than the [±cons] tiers.

The Jardine and McMullin 2017 model (TSLIA), as well as Burness and McMullin (2019, 2021)'s model, were proposed primarily for proving learnability results about tier-local generalizations. Such proofs require certain assumptions about the data available to the learner. These conditions specify a *characteristic sample*, which, as discussed by Jardine and McMullin (2017:75), may not be satisfied in natural language data (e.g., due to interaction with other phonotactic constraints). TSLIA's performance suggests that such a sample is indeed not present in these datasets. Nevertheless, as discussed in section 3.2, these models share with D2L the process of iterative deletion of segments from the tier. Thus, D2L encodes the spirit of these models, while using featural representations to generalize over classes of segments, and the Tolerance Principle to deal with the sparsity and exceptions inevitable in naturalistic data.

The LSTM neural network model failed to match human behavior, as it generalized harmony/dissimilation to all types of test instances in the artificial language experiments, including those where harmony is blocked for humans by intervening segments. This suggests that LSTM may not be well-suited for capturing blockers. Moreover, LSTM appears to be overly sensitive to the baseline frequency of an alternating segment's surface forms.

In the Latin data, the most frequent form of /L/ is [l], and 100% of LSTM's errors result from producing /L/ → *[l] instead of [r]. By far the most common error is producing the most frequent surface form of an alternating segment in Turkish and Finnish as well.

6.2 Future Directions

In its current form, D2L learns categorical rules. Extending D2L to handle variation is an important step for future research. This will likely involve the current model for constructing the structural description of rules, but would need to allow them to be probabilistic (Labov, 1969; Mayer, 2021).

This paper considered noninteracting (dis)harmony processes. A model like the *Parsimonious Local Phonology* learner from Belth (To appear) provides a broader framework for how D2L could fit in a larger theory of phonological learning that includes epenthesis, deletion, and multiple generalizations.

Another direction for future research involves investigating whether metrical stress and tonal patterns can be learned by a similar process. For example, Jardine (2016a) argued that while most of segmental phonology can be characterized in terms of tier-locality, there are numerous processes in tonal phonology (e.g. tonal plateauing) that may require greater expressive power (unbounded circumambient) to account for. Moreover, McCollum *et al.* (2020) argued that Tutrugbu ATR harmony requires the same expressive power as Jardine's tonal analyses, suggesting that unbounded circumambient processes may be more prevalent in segmental phonology than previously thought. The key characteristic of unbounded circumambient phenomena is the involvement of dependencies that are arbitrarily far away in *both directions*. Thus, future research could investigate whether D2L's approach of iteratively deleting adjacent, unuseful, segments could render local the relevant dependencies from both directions.

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Notes

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[1]For ease of exposition, I use [+sib] to refer to a set of sibilants in a language's segment inventory, not as a commitment a universal feature [+sib] with that extension. The experiments do not use a [+sib] feature and can only reference sibilants via a [stri] feature.

[2]To learn noniterative processes, the model can straight-forwardly be extended by applying rules simultaneously instead.

[3]Alternatively, if only alternating vowels are underspecified, [−high] vowels would be fully specified underlyingly and thus excluded automatically by the [?round] condition.

[4]The default form of /S/ is [s]; I discuss how D2L discovers this in section 2.3.3.

[5]For ease of exposition, I consider only natural classes describable with a single feature, but clearly the same process could apply over natural classes requiring more features to specify.

[6]This is similar to the strategy of Minimal Generalization described by Albright and Hayes (2002, 2003), which constructs a natural class for a set of segments by retaining all features shared by those segments in order to include as few segments outside the set as possible. My method differs in that it does not allow segments in the target set A to be swallowed by the deletion set D, since doing so would prevent the target segments from being targeted.

[7]Since many processes involve all segments on the tier, once a successful rule is constructed, if the context set C_l/C_r can be set to equal the tier T without changing the accuracy of the rule, then this is done.

[8]https://github.com/gouskova/inductive_projection_learner

[9]I chose to report the human results in this way because Finley (2011) only reported these conclusions, not the actual mean rates of the harmonizing choice.

[10]In practice, I found that the exposure dataset was small enough that GG did not always pick up on these regularities.